ROTATIONAL RELAXATION IN A FREELY EXPANDING NITROGEN JET

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UDC 533.6.011.8

In a supersonic outflow of gas into a vacuum, the local frequencies of collisions between molecules decrease rapidly in the downstream direction, which leads to disturbance of the equilibrium between the translational and internal degrees of freedom. The rotational relaxation in the region of free expansion of a supersonic, incompletely expanded jet of nitrogen on the assumption of negligible effect of relaxation on the translational temperature and Mach number was examined in [1]. A comparison of the results of calculations [1] with experimental data for a sonic nozzle [2] revealed a considerable disagreement. In the present paper the problem of free axisymmetric expansion of nitrogen with due regard to the effect of rotational relaxation on the gasdynamic parameters is solved numerically. For the calculation, we use the method of characteristics in the form proposed in [3].

1. Main Assumptions

The free expansion of nitrogen from a round nozzle at moderate temperatures, when the internal energy of the gas is composed of the energy of translational and rotational degrees of freedom, is considered. The following assumptions are made in the calculation:

1) the effect of viscosity and thermal conduction on the flow parameters are negligible [4];

2) the energy distribution of rotational degrees of freedom corresponds to a Boltzmann distribution, which allows the introduction of a rotational temperature T_r;

3) the rotational relaxation is represented by a relaxation equation [5] of the form

$$\frac{dT_r}{dt} = \frac{T - T_r}{\tau_r}$$

$$\tau_r = Z\tau, \ \tau = \left[\sqrt{2}\,\sigma N\left(\frac{8kT}{\pi m}\right)^{1/2}\right]^{-1}$$

Here, T is the translational temperature; τ_r is the rotational relaxation time; τ is the mean time of the mean free path; Z is the number of collisions required for establishment of equilibrium between the rotational and translational degrees of freedom; σ is the collision cross section; N is the concentration; and m is the mass of the molecule.

For nitrogen ultrasonic measurements at $T \approx 300$°K give $Z \approx 5$ [5]. The collision cross section σ can be obtained from experimental viscosity data. In the temperature range $50 < T < 300$°K the viscosity μ of nitrogen is approximated satisfactorily by the relationship $\mu \sim T$ [6, 7], which corresponds to $\sigma \sim T^{-1/2}$. In the calculation the expression $\sigma(T) = 4.4 \cdot 10^{-15}\sqrt{300/T}$ (cm²) was used.

2. System of Equations and Calculation Procedure

In view of the assumptions made, the system of equations for calculating the parameters of nonviscous, non-heat-conducting, rotationally relaxing nitrogen in a cylindrical coordinate system has the form

$$\frac{\partial (y\rho u)}{\partial x} + \frac{\partial (y\rho v)}{\partial y} = 0 \tag{2.1}$$

Leningrad. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 176-178, May-June, 1972. Original article submitted October 6, 1971.

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{1}{\rho}\frac{\partial p}{\partial x}=0 \quad (2.2)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{1}{\rho}\frac{\partial p}{\partial y}=0 \quad (2.3)$$

$$h(T,T_r)+{}^1/_2w^2=h_0,\ w^2=u^2+v^2 \quad (2.4)$$

$$h(T,T_r)={}^5/_2RT+RT_r \quad (2.5)$$

$$p=\rho RT \quad (2.6)$$

$$\frac{dT_r}{dt}=\frac{T-T_r}{\tau_r} \quad (2.7)$$

Here, x and y are coordinates (x is directed along the axis of symmetry); ρ is the density; p is the pressure; w, u, and v are the velocity and its projections on the x and y axis; h is the enthalpy; R is the gas constant.

The system of equations (2.1)-(2.7) can be solved by the method of characteristics [3]. The theoretical system of equations includes equations of the characteristics of the first and second families and the relationships along the stream lines. The equations of the characteristics are written in the form

$$dx_+=\frac{\beta\mp\zeta}{\beta\zeta\pm1}dy_+ \quad (2.8)$$

$$\frac{1}{1+\zeta^2}d\zeta\pm\frac{\beta}{\rho_+w_+^2}dp_+\pm\frac{1}{\beta\zeta\pm1}\left[\frac{\zeta}{y_+}+\frac{(1+\zeta^2)^{1/2}(1-T_{r+}/T_+)}{{}^5/_2w_a\tau_rw_+}\right]dy_+=0 \quad (2.9)$$

$$d\psi_+=\pm\frac{\rho_+w_+y_+(1+\zeta^2)^{1/2}}{\beta\zeta\pm1}dy_+ \quad (2.10)$$

Here ψ is the stream functions, $\zeta=\tan\theta$ (θ is the angle of inclination of the velocity vector to the x axis), $\beta=\sqrt{w^2/a^2-1}$, $a^2=1.67$ RT, a is the "frozen" velocity of sound, $x_+=x/r_a$, $y_+=y/r_a$, r_a is the radius of the nozzle exit section

$$\rho_+=\rho/\rho_a,\ w_+=w/w_a,\ p_+=p/\rho_aw_a^2,\ \psi_+=\psi/\rho_aw_ar_a^2,\ T_+=T/T_a,\ T_{r+}=T_r/T_a$$

The subscript a refers to parameters at the nozzle section. The relationships along the stream lines have the form

$$dy_+=\zeta dx_+ \quad (2.11)$$

$$dT_{r+}=\frac{r_a(1+\zeta^2)^{1/2}(T_+-T_{r+})}{w_a\tau_rw_+}dx_+ \quad (2.12)$$

$$dT_+-\frac{2}{5}\frac{w_a^2}{RT_a}\frac{dp_+}{\rho_+}+\frac{2}{5}dT_{r+}=0 \quad (2.13)$$

$$\frac{w_a^2}{RT_a}\frac{w_+^2}{2}+\frac{5}{2}T_++T_{r+}=\frac{7}{2}T_0 \quad (2.14)$$

The parameters at the nozzle section are essential for the calculation. Equations (2.8)-(2.14) were written in finite-difference form in accordance with [3]. The computer program included subroutines for calculating the parameters on the initial surface, in the flow field, at the nozzle section, and on the axis of symmetry.

3. Results of Calculation and Their Analysis

The system of equations (2.8)-(2.14) was used to calculate the free expansion of nitrogen in conditions corresponding to the experiment in [2] ($M_a=w_a/a_a=1$, $r_a=5$ mm, temperature in receiver $T_0=300$°K, $p_0r_a=7.5$ and 240 torr · mm, p_0 is the pressure in the receiver). The calculation was carried out for

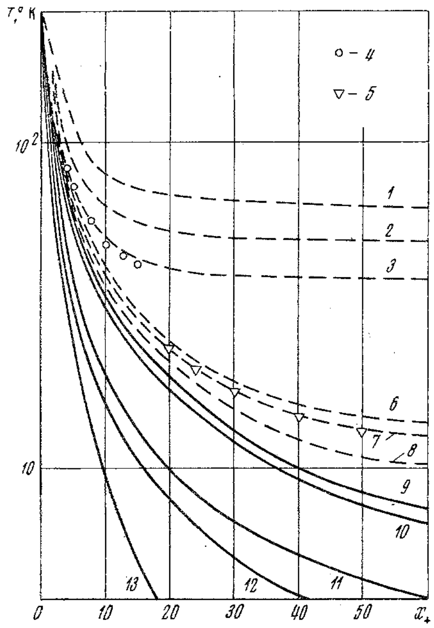

Fig. 1

Z = 5 and 10. The difficulty entailed in calculation close to the surface M = 1 was circumvented by choosing a surface with equal parameters, corresponding to M = 1.1, as the initial surface. It was assumed that when M = 1.1 the flow is an equilibrium flow. The number of points taken on the initial surface was n = 25. The value of T_{r+} on the initial surface was determined from the condition

$$\frac{dT_{r+}}{dx_+} = \frac{dT_+}{dx_+} \quad \text{for } y_+ = 0,\ x_+ = x_{1+}$$

where x_{1+} is the value of x_+ at the point of intersection of the initial surface with the axis of symmetry. A value $\Delta\beta = 2 \cdot 10^{-2}$ was taken for the pitch of the stream turn at the nozzle exit. A check calculation showed that when $\Delta\beta$ was reduced and n doubled, M and ψ were altered by less than 1% in comparison with the indicated values.

Figure 1 shows the results of calculation of T_r [dashed curves: 1) $p_0 r_a$ = 7.5 torr · mm, Z = 10; 3) $p_0 r_a$ = 7.5 torr · mm, Z = 5; 7) $p_0 r_a$ = 240 torr · mm, Z = 10; 8) $p_0 r_a$ = 240 torr · mm, Z = 5] and T [solid curves; 10) $p_0 r_a$ = 240 torr · mm, Z = 10; 11) $p_0 r_a$ = 7.5 torr · mm, Z = 5; 12) $p_0 r_a$ = 7.5 torr · mm, Z = 10] on the axis of symmetry and also the results of calculation of T_r [1] with similar initial data [2) $p_0 r_a$ = 7.5 torr · mm, Z = 5; 6) $p_0 r_a$ = 240 torr · mm, Z = 10], but without taking into account the effect of rotational relaxation on the flow geometry and gasdynamic parameters. The figure also gives the results of measurements [2] [4) $p_0 r_a$ = 7.5 torr · mm; 5) $p_0 r_a$ = 240 torr · mm) and the values of T for cases of isentropic expansion of gas with γ = 1.4 (curve 9) and γ = 1.67 (curve 13)].

A reduction of $p_0 r_a$, as was to be expected, leads to an earlier disturbance of equilibrium between the translational and rotational degrees of freedom. Allowance for the effect of rotational relaxation on the expansion of nitrogen greatly reduces the disagreement between the results of calculation [1] and measurement [2] of the rotational temperature. The reduction of T with the freezing of T_r leads, in accordance with (2.7), to a more rapid reduction of T_r. When $p_0 r_a$ = 7.5 torr · mm a better agreement between the calculation and experimental results is obtained with Z = 5. At $p_0 r_a$ = 240 torr · mm the slight deviation from equilibrium does not allow a choice between Z = 5 or 10.

Freezing of the rotational temperature increases the rate of cooling of the gas in translational degrees of freedom. The change in T, however, after disturbance of equilibrium does not correspond to γ = 1.67, which would result from complete freezing of the rotations. The occurrence of rare collisions at $T_r \gg T$ is sufficient for significant "replenishment" of the translational degrees of freedom, since the energy transmitted in one collision is proportional to $T_r - T$. Hence, the effective value of γ is close to the equilibrium value. Energy transfer between the rotational and translational degrees of freedom due to the rare collisions has no significant effect on the variation of T_r.

It should be noted in conclusion that the experimental data of [2] indicate that the population of the upper rotational levels does not conform with a Boltzmann distribution. The observed nonequilibrium of the populations can be attributed to the penetration of "hotter" molecules of the environment into the free-expansion region [8]. The value of T_r used in the comparison of the results of calculation and experiment was that obtained in [2] from the relative population of the lower rotational levels, where the effect of environmental molecules diffusing into the jet is insignificant. An accurate assessment of the role of this factor will require experimental data for the populations of the rotational levels in the free expansion region of the jet for different pressures in the surrounding atmosphere.

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